## PUNE VIDYARTHI GRIHA's COLLEGE OF ENGINEERING, NASHIK.

${ }^{*}$ CODE OPTIMIZATION "

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## CONTENTS :

Need for Optimization, local, global and loop optimization, Optimizing transformations, compile time evaluation, common sub-expression elimination, variable propagation, code movement, strength reduction, dead code elimination, DAG based local optimization, Introduction to global data flow analysis, Data flow equations and iterative data flow analysesis.

## Code Optimization :

- A main goal is to achieve a better performance



## What is optimization?

In computing, optimization is the process of modifying a system to make some aspect of it work more efficiently or use fewer resources.

For instance, a computer program may be optimized so that it executes more rapidly, or is capable of operating with less memory storage or other resources, or draw less power. The system may be a single computer program, a collection of computers or even an entire network such as the internet.

## TYPES OF OPTIMIZATION

Optimization is of two types

1. Machine Dependent optimization
2. Machine independent
$\square$ The machine dependent optimization is based on characteristics of the target machine (the instruction set used and addressing modes used for the instructions) to generate efficient target code.
$\square$ The machine independent optimization is based on the characteristics of the programming languages for appropriate programming structure and usage of efficient anrifiniotic properties in in ofderto reduce executiontime

## Code Optimization Techniques

Constant propagation
If the value of a variable is a constant, then replace the variable by the constant

It is not the constant definition, but a variable is assigned to a constant
The variable may not always be a constant
E.g.

$$
\begin{aligned}
& \mathrm{N}:=10 ; \mathrm{C}:=2 \\
& \text { for }(\mathrm{i}:=0 ; \mathrm{i}<\mathrm{N} ; \mathrm{i}++)\left\{\mathrm{s}:=\mathrm{s}+\mathrm{i}^{*} \mathrm{C} ;\right\} \\
& \quad \Rightarrow \text { for }(\mathrm{i}:=0 ; \mathrm{i}<10 ; \mathrm{i}++)\left\{\mathrm{s}:=\mathrm{s}+\mathrm{i}^{*} 2 ;\right\}
\end{aligned}
$$

If (C) go to $\ldots \Rightarrow$ go to ...
The other branch, if any, can be eliminated by other optimizations
Requirement:
After a constant assignment to the variable

## Code Optimization Techniques Constant folding

In a statement $\mathrm{x}:=\mathrm{y}$ op z or $\mathrm{x}:=\mathrm{op} \mathrm{y}$
If y and z are constants
Then the value can be computed at compilation time
Example
\#define M 10

$$
\begin{aligned}
& x:=2 * M \Rightarrow x:=20 \\
& \text { If }(M<0) \text { goto } L \Rightarrow \text { can be eliminated } \\
& y:=10 * 5 \Rightarrow y:=50
\end{aligned}
$$

Difference: constant propagation and folding
Propagation: only substitute a variable by its assigned constant
Folding: Consider variables whose values can be computed at compilation time
and controls whose decision can be determined at compilation time

## Code Optimization Techniques

## Algebraic simplification

More general form of constant folding, e.g.,

$$
\begin{aligned}
& \mathrm{x}+0 \Rightarrow \mathrm{x} \quad \mathrm{x}-0 \Rightarrow \mathrm{x} \\
& \mathrm{x} * 1 \Rightarrow \mathrm{xx} / 1 \Rightarrow \mathrm{x} \\
& \mathrm{x} * 0 \Rightarrow 0
\end{aligned}
$$

Repeatedly apply the rules

$$
(y * 1+0) / 1 \Rightarrow y
$$

## Strength reduction

Replace expensive operations

$$
\text { E.g., } x:=x * 8 \Rightarrow x:=x \ll 3
$$

## Code Optimization Techniques <br> Copy propagation

Extension of constant propagation
After y is assigned to x , use y to replace x till x is assigned again
Example

$$
\begin{aligned}
& x:=y ; \quad \Rightarrow \quad s:=y * f(y) \\
& s:=x * f(x)
\end{aligned}
$$

Reduce the copying
If $y$ is reassigned in between, then this action cannot be performed

## Code Optimization Techniques <br> Common subexpression elimination

Example:

$$
\begin{aligned}
& \mathrm{a}:=\mathrm{b}+\mathrm{c} \\
& \mathrm{c}:=\mathrm{b}+\mathrm{c} \\
& \mathrm{~d}:=\mathrm{b}+\mathrm{c}
\end{aligned} \quad \Rightarrow \quad \begin{aligned}
& \mathrm{a}:=\mathrm{b}+\mathrm{c} \\
& \mathrm{c}:=\mathrm{a} \\
& \mathrm{~d}:=\mathrm{b}+\mathrm{c}
\end{aligned}
$$

Example in array index calculations
$c[i+1]:=a[i+1]+b[i+1]$
During address computation, $\mathrm{i}+1$ should be reused Not visible in high level code, but in intermediate code

## Code Optimization Techniques Unreacheable code elimination

Construct the control flow graph
Unreachable code block will not have an incoming edge
After constant propagation/folding, unreachable branches can be eliminated
Dead code elimination
Ineffective statements

$$
\begin{array}{ll}
x:=y+1 & \text { (immediately redefined, eliminate!) } \\
y:=5 & \Rightarrow \quad y:=5 \\
x:=2 * z & x:=2 * z
\end{array}
$$

A variable is dead if it is never used after last definition
Eliminate assignments to dead variables
Need to do data flow analysis to find dead variables

## Code Optimization Techniques

## Loop optimization Techniques

- Loop invariant detection and code motion
- Induction variable elimination
- Strength reduction in loops
- Loop unrolling
- Loop peeling
- Loop fusion


## Code Optimization Techniques

Loop invariant detection and code motion
If the result of a statement or expression does not change within a loop, and it has no external side-effect
Computation can be moved to outside of the loop
Example
for ( $\mathrm{i}=0$; $\mathrm{i}<\mathrm{n} ; \mathrm{i}++$ ) $\mathrm{a}[\mathrm{i}]:=\mathrm{a}[\mathrm{i}]+\mathrm{x} / \mathrm{y}$;
Three address code

$$
\text { for }(\mathrm{i}=0 ; \mathrm{i}<\mathrm{n} ; \mathrm{i}++)\{\mathrm{c}:=\mathrm{x} / \mathrm{y} ; \mathrm{a}[\mathrm{i}]:=\mathrm{a}[\mathrm{i}]+
$$

$$
\begin{aligned}
& c ;\} \\
& \Rightarrow c:=x / y \\
& \quad \text { for }(i=0 ; i<n ; i++) a[i]:=a[i]+c
\end{aligned}
$$

## Code Optimization Techniques

Strength reduction in loops
Example

$$
\begin{aligned}
& \mathrm{s}:=0 ; \text { for }(\mathrm{i}=0 ; \mathrm{i}<\mathrm{n} ; \mathrm{i}++)\{\mathrm{v}:=4 * i ; \mathrm{s}:=\mathrm{s}+\mathrm{v} ;) \\
& \Rightarrow \mathrm{s}:=0 ; \text { for }(\mathrm{i}=0 ; \mathrm{i}<\mathrm{n} ; \mathrm{i}++)\{\mathrm{v}:=\mathrm{v}+4 ; \mathrm{s}:=\mathrm{s}+\mathrm{v} ;)
\end{aligned}
$$

## Induction variable elimination

If there are multiple induction variables in a loop, can eliminate the ones which are used only in the test condition
Example
$\mathrm{s}:=0$; for $(\mathrm{i}=0 ; \mathrm{i}<\mathrm{n} ; \mathrm{i}++)\{\mathrm{s}:=4 * \mathrm{i} ; \ldots\}-\mathrm{i}$ is not referenced in loop
$\Rightarrow \mathrm{s}:=0 ; \mathrm{e}:=4^{*} \mathrm{n}$; while $(\mathrm{s}<\mathrm{e})\{\mathrm{s}:=\mathrm{s}+4 ;\}$

## Code Optimization Techniques

## Loop unrolling

Execute loop body multiple times at each iteration
Get rid of the conditional branches, if possible
Allow optimization to cross multiple iterations of the loop
Especially for parallel instruction execution
Space time tradeoff
Increase in code size, reduce some instructions

## Loop peeling

Similar to unrolling
But unroll the first and/or last few iterations

## Loop fusion

## Example

for $\mathrm{i}=1$ to N do

$$
A[i]=B[i]+1
$$

endfor
for $\mathrm{i}=1$ to N do $\mathrm{C}[\mathrm{i}]=\mathrm{A}[\mathrm{i}] / 2$
endfor
for $\mathrm{i}=1$ to N do $D[i]=1 / C[i+1]$
endfor


Before Loop Fusion

## Principle sources of optimization

- Local optimization: within a basic block
- Global optimization: otherwise
- Mixed


## PRINCIPLE SOURCES OF OPTIMIZATION

- Function Preserving Transformations :
i) Common Subexpression Elimination
ii)Copy Propogation
iii)Dead - code Elimination
iv)Constant folding
v)Loop Optimization : Code Motion, Strength Reduction, Induction Variable Elimination


## Classic Examples of Local and Global Code Optimizations

- Local
- Constant folding
- Constant combining
- Strength reduction
- Constant propagation
- Common subexpression elimination
- Backward copy propagation
- Global
- Dead code elimination
- Constant propagation
- Forward copy propagation
- Common subexpression elimination
- Code motion
- Loop strength reduction
- Induction variable elimination


## Local: Constant Folding



- Goal: eliminate unnecessary operations
- Rules:

1. X is an arithmetic operation
2. If $\operatorname{src} 1(\mathrm{X})$ and $\operatorname{src} 2(\mathrm{X})$ are constant, then change X by applying the operation

## Local: Constant Combining



- Goal: eliminate unnecessary operations
- First operation often becomes dead after constant combining
- Rules:

1. Operations X and Y in same basic block
2. $X$ and $Y$ have at least one literal src
3. Y uses dest(X)
4. None of the srcs of $X$ have defs between X and Y (excluding Y)

## Local: Strength Reduction



- Goal: replace expensive operations with cheaper ones
- Rules (common):

1. X is an multiplication operation where $\operatorname{src} 1(\mathrm{X})$ or $\operatorname{src} 2(\mathrm{X})$ is a const $2^{k}$ integer literal
2. Change $X$ by using shift operation
3. For $k=1$ can use add

## Local: Constant Propagation

```
r1 = 5
r2 = m
r4 = r4 + r1 -... r4 = r4 + 5
r1 = r1 + r2 ..... r1 = 5 +_x
r1 = r1 + 1 {... r1 = 5 +_x + 1
r3 = 12
r8 = r1 - r2 &... r8 = 5 + _x + 1 - _x
r9 = r3 + r54... r9 = 12 +r5
r3 = r2 + 1 &.... r3 = _x + 1
r7 = r3 - r1 {... r7 = _x + 1 - 5 - _x - 1
```

- Goal: replace register uses with literals (constants) in a single basic block
Rules:

1. Operation $X$ is a move to register with $\operatorname{src} 1(\mathrm{X})$ literal
2. Operation Y uses $\operatorname{dest}(\mathrm{X})$
3. There is no $\operatorname{def}$ of $\operatorname{dest}(\mathrm{X})$ between X and Y (excluding defs at X and Y )
4. Replace dest(X) in $Y$ with $\operatorname{src} 1(\mathrm{X})$

## Local: Common Subexpression Elimination (CSE)

$$
\begin{aligned}
& r 1=r 2+r 3 \\
& r 4=r 4+1 \\
& r 1=6 \\
& r 6=r 2+r 3 \\
& r 2=r 1-1 \\
& r 5=r 4+1 \\
& r 7=r 2+r 3 \\
& r 5=r 1-1
\end{aligned}
$$

- Goal: eliminate recomputations of an expression
- More efficient code
- Resulting moves can get copy propagated (see later)
- Rules:

1. Operations X and Y have the same opcode and Y follows X
2. $\operatorname{src}(\mathrm{X})=\operatorname{src}(\mathrm{Y})$ for all $\operatorname{srcs}$
3. For all srcs, no def of a src between X and Y (excluding Y)
4. No def of $\operatorname{dest}(X)$ between $X$ and Y (excluding X and Y )
5. Replace Y with move $\operatorname{dest}(\mathrm{Y})$ $=\operatorname{dest}(X)$

## Local: Backward Copy Propagation

```
r1 = r8 + r9
r2 = r9 + r1
r4 = r2
r6 = r2 + 1 &... r7 = r2 + 1
r9 = r1
r7 = r6 s... remove r7 = r6
r5 = r7 + 1
r4 = 0
r8 = r2 + r7
```

- Goal: propagate LHS of moves backward
- Eliminates useless moves
- Rules (dataflow required)

1. $X$ and $Y$ in same block
2. Y is a move to register
3. $\operatorname{dest}(\mathrm{X})$ is a register that is not live out of the block
4. Y uses $\operatorname{dest}(\mathrm{X})$
5. $\quad \operatorname{dest}(\mathrm{Y})$ not used or defined between X and Y (excluding X and Y )
6. No uses of dest( X ) after the first redef of dest(Y)
7. Replace $\operatorname{src}(\mathrm{Y})$ on path from X to $Y$ with $\operatorname{dest}(X)$ and remove $Y$

## Global: Dead Code Elimination



- Goal: eliminate any operation who's result is never used
- Rules (dataflow required)

1. X is an operation with no use in def-use (DU) chain, i.e. $\operatorname{dest}(\mathrm{X})$ is not live
2. Delete X if removable (not a mem store or branch)

- Rules too simple!
- Misses deletion of $r 4$, even after deleting $r 7$, since $r 4$ is live in loop
- Better is to trace UD chains backwards from "critical" operations


## Global: Constant Propagation



- Goal: globally replace register uses with literals
- Rules (dataflow required)

1. X is a move to a register with $\operatorname{src} 1(\mathrm{X})$ literal
2. $Y$ uses $\operatorname{dest}(\mathrm{X})$
3. $\operatorname{dest}(\mathrm{X})$ has only one def at X for use-def (UD) chains to Y
4. Replace $\operatorname{dest}(\mathrm{X})$ in $Y$ with $\operatorname{src} 1(\mathrm{X})$

## Global: Forward Copy Propagation



- Goal: globally propagate RHS of moves forward
- Reduces dependence chain
- May be possible to eliminate moves
- Rules (dataflow required)

1. X is a move with $\operatorname{src} 1(\mathrm{X})$ register
2. Y uses $\operatorname{dest}(\mathrm{X})$
3. $\operatorname{dest}(\mathrm{X})$ has only one def at X for UD chains to $Y$
4. $\operatorname{src} 1(X)$ has no def on any path from X to Y
5. Replace $\operatorname{dest}(\mathrm{X})$ in Y with $\operatorname{src} 1(\mathrm{X})$

## Global: Common Subexpression Elimination (CSE)



- Goal: eliminate recomputations of an expression
- Rules:

1. $X$ and $Y$ have the same opcode and X dominates Y
2. $\operatorname{src}(X)=\operatorname{src}(Y)$ for all srcs
3. For all srcs, no def of a src on any path between X and Y (excluding Y)
4. Insert $r x=\operatorname{dest}(X)$ immediately after X for new register rx
5. Replace Y with move $\operatorname{dest}(\mathrm{Y})$

$$
=r x
$$

## Global: Code Motion



- Goal: move loop-invariant computations to preheader
- Rules:

1. Operation X in block that dominates all exit blocks
2. X is the only operation to modify $\operatorname{dest}(\mathrm{X})$ in loop body
3. All srcs of $X$ have no defs in any of the basic blocks in the loop body
4. Move X to end of preheader
5. Note 1 : if one src of X is a memory load, need to check for stores in loop body

## Global: Loop Strength Reduction



Replace expensive cROF. ANAND GHARU with induction variables

## Global: Induction Variable Elimination



Replace induction variable in expressions with another

## The Code Optimizer

- Control flow analysis
- Data-flow analysis
- Transformations



## Determining Loops in Flow Graphs: Dominators

- Dominators: d dom $n$
- Node $d$ of a CFG dominates node $n$ if every path from the initial node of the CFG to $n$ goes through $d$
- The loop entry dominates all nodes in the loop
- The immediate dominator $m$ of a node $n$ is the last dominator on the path from the initial node to $n$
- If $d \neq n$ and $d$ dom $n$ then $d$ dom $m$


## Dominator Trees



CFG


Dominator tree

## Natural Loops

- A back edge is is an edge $a \rightarrow b$ whose head $b$ dominates its tail $a$
- Given a back edge $n \rightarrow d$
- The natural loop consists of $d$ plus the nodes that can reach $n$ without going through $d$
- The loop header is node $d$
- Unless two loops have the same header, they are disjoint or one is nested within the other
- A nested loop is an inner loop if it contains no other loops


## Natural Inner Loops Example



CFG
Dominator tree

## Natural Outer Loops Example

Natural loop for $1 \operatorname{dom} 9$

CFG


Dominator tree

## Pre-Headers

- To facilitate loop transformations, a compiler often adds a preheader to a loop
- Code motion, strength reduction, and other loop transformations populate the preheader



## Data Flow Analysis

Data flow analysis is used to collect information about the flow of data values across basic blocks.
-Dominator analysis collected global information regarding the program's structure
-For performing global code optimizations global information must be collected regarding values of program variables.

- Local optimizations involve statements from same basic block
- Global optimizations involve statements from different basic blocks $\rightarrow$ data flow analysis is performed to collect global information that drives global optimizations


## Local and Global Optimization

$x=A+B$
$\vdots$

$y=A+B$$\longrightarrow$| $x=A+B$ |
| :---: |
| $\vdots$ |
| $y=x$ |

hocal optimization


Global optimization

Applications of Data Flow

## Analysis

- Applicability of code optimizations
- Symbolic debugging of code
- Static error checking
- Type inference


## Global Data Flow Analysis

- These Analyze following properties :

1. Reaching Definitions : Constant Propagation, Dead code Elimination, copy propogation
2. Available Expressions : Common Subexpression Elimination
3. Live Variable Analysis : Register Allocation
4. Very Busy Expressions : Dead code Elimination, Register Allocation

## Data Flow Equation

Data Flow Analysis is done by the equation :

$$
\operatorname{Out}[\mathrm{S}]=\operatorname{gen}[\mathrm{S}] \mathrm{U}(\mathrm{in}[\mathrm{~S}]-\operatorname{kill}[\mathrm{S}])
$$

i.e information at the end is either generated within the statement or at the beginning and not killed as control flows through the statement

## 1. Reaching Definitions

Definition $d$ of variable $v$ : a statement $d$ that assigns a value to v .

Use of variable $v$ : reference to value of $v$ in an expression evaluation.

Definition $d$ of variable $v$ reaches a point $p$ if there exists a path from immediately after $d$ to $p$ such that definition $d$ is not killed along the path.
Definition d is killed along a path between two points if there exists an assignment to variable v along the path.

## Example


d reaches u along path $h_{2}$ d does not reach $u$ along path ${ }_{1}$
Since there exists a path from d to u along which dis not killed (i.e., path ${ }_{2}$ ), d reaches u.

## Computing Reaching Definitions

At each program point $p$, we compute the set of definitions that reach point $p$.
Reaching definitions are computed by solving a system of equations (data flow equations).


## Data Flow Equations

IN[B]: Definitions that reach B's entry. OUT[B]: Definitions that reach B's exit.

$$
\begin{aligned}
& \operatorname{IN}[B]= \cup \operatorname{OUT}(p) \\
& \operatorname{pepred}(B) \\
& \operatorname{OUT}(B)= \operatorname{GEN}(B) \cup(\operatorname{NN}(B)-\operatorname{KILL}(B))
\end{aligned}
$$

GEN[B]: Definitions within $B$ that reach the end of $B$.
KILL[B]: Definitions that never reach the end of $B$ due to redefinitions of variables in $B$.

## Reaching Definitions Contd.

- Forward problem - information flows forward in the direction of edges.
- May problem - there is a path along which definition reaches a point but it does not always reach the point.
Therefore in a May problem the meet operator is the Union operator.


## Applications of Reaching

 Definitions- Constant Propagation/folding

- Copy Propagation



## 2. Available Expressions

An expression is generated at a point if it is computed at that point.
An expression is killed by redefinitions of operands of the expression.

An expression $\mathrm{A}+\mathrm{B}$ is available at a point if every path from the start node to the point evaluates $\mathrm{A}+\mathrm{B}$ and after the last evaluation of $A+B$ on each path there is no redefinition of either $A$ or $B$ (i.e., $A+B$ is not killed).

## Available Expressions



Available expressions problem computes: at each program point the set of expressions available at that point.

## Data Flow Equations

IN[B]: Expressions available at B's entry. OUT[B]: Expressions available at B's exit.

$$
\begin{aligned}
\operatorname{IN}[B]= & \cap \operatorname{OUT}(p) \\
& p \in \operatorname{pred}(B) \\
\operatorname{OUT}[B]= & \operatorname{GEN}[B] \cup(\operatorname{IN}[B]-\operatorname{KILL}[B])
\end{aligned}
$$

GEN[B]: Expressions computed within B that are available at the end of $B$.
KILL[B]: Expressions whose operands are redefined in B.

## Available Expressions Contd.

- Forward problem - information flows forward in the direction of edges.
- Must problem - expression is definitely available at a point along all paths.
Therefore in a Must problem the meet operator is the Intersection operator.


# Applications of Available 

## Expressions

- Common Subexpression Elimination



## 3. Live Variable Analysis

A path is X -clear is it contains no definition of X .
A variable $X$ is live at point $p$ if there exists a $X$-clear path from $p$ to a use of $X$; otherwise $X$ is dead at $p$.


Live Variable Analysis Computes:

At each program point $p$ identify the set of variables that are live at $p$.

## Data Flow Equations

IN[B]: Variables live at B's entry.
OUT[B]: Variables live at B's exit.

$$
\begin{aligned}
\operatorname{OUT}[B]= & \bigcup \operatorname{IN}[S] \\
& S \in \operatorname{Succ}(B) \\
\operatorname{IN}[B]= & \operatorname{GEN}[B] \cup \operatorname{OUT}[B]-\operatorname{KILL[B]})
\end{aligned}
$$

GEN[B]: Variables that are used in B prior to their definition in B.
KILL[B]: Variables definitely assigned value in $B$ before any use of that variable in $B$.

## Live Variables Contd.

- Backward problem - information flows backward in reverse of the direction of edges.
- May problem - there exists a path along which a use is encountered.

Therefore in a May problem the meet operator is the Union operator.

## Applications of Live Variables

- Register Allocation
- Dead Code Elimination

- Code Motion Out of Loops



## 4. Very Busy Expressions

A expression $A+B$ is very busy at point $p$ if for all paths starting at p and ending at the end of the program, an evaluation of $A+B$ appears before any definition of $A$ or $B$.


Compute for each program point the set of very busy expressions at the point.

## Data Flow Equations

IN[B]: Expressions very busy at B's entry. OUT[B]: Expressions very busy at B's exit.

$$
\begin{aligned}
\operatorname{OUT}[B]= & \bigcap_{\operatorname{S\in Suc}(B)} \operatorname{IN}[S] \\
\operatorname{NN}[B]= & \operatorname{GEN}[B] \cup(\operatorname{OUT}[B]-\operatorname{KILL}[B])
\end{aligned}
$$

GEN[B]: Expression computed in $B$ and variables used in the expression are not redefined in B prior to expression's evaluation in $B$.
KILL[ $B]$ : Expressions that use variables that are

## Very Busy Expressions Contd.

- Backward problem - information flows backward in reverse of the direction of edges.
- Must problem - expressions must be computed along all paths.

Therefore in a Must problem the meet operator is the Intersection operator.

## Summary

|  | May/Union | Must/Intersecti <br> on |
| :--- | :---: | :---: | :---: |
| Forward | Reaching <br> Definitions | Available <br> Expressions |
| Backward | Live <br> Variables | Very Busy <br> Expressions |

## Reaching Definitions



Then, the data-flow equations for $S$ are:

$$
\begin{array}{ll}
\operatorname{gen}[S] & =\{d\} \\
\operatorname{kill}[S] & =D_{\mathrm{a}}-\{d\} \\
\operatorname{out}[S] & =\operatorname{gen}[S] \cup(\operatorname{in}[S]-\operatorname{kill}[S])
\end{array}
$$

## Reaching Definitions


is of the form


$$
\begin{array}{ll}
\operatorname{gen}[S] & =\operatorname{gen}\left[S_{2}\right] \cup\left(\operatorname{gen}\left[S_{1}\right]-\operatorname{kill}\left[S_{2}\right]\right) \\
\operatorname{kill}[S] & =\operatorname{kill}\left[S_{2}\right] \cup\left(\operatorname{kill}\left[S_{1}\right]-\operatorname{gen}\left[S_{2}\right]\right) \\
\operatorname{in}\left[S_{1}\right] & =\operatorname{in}[S] \\
\operatorname{in}\left[S_{2}\right] & =\operatorname{out}\left[S_{1}\right] \\
\operatorname{out}[S] & \text { PROF:AUAUtDS} \left.S_{2}\right] \text { ARU }
\end{array}
$$

## Reaching Definitions


is of the form


$$
\begin{array}{ll}
\operatorname{gen}[S] & =\operatorname{gen}\left[S_{1}\right] \cup \operatorname{gen}\left[S_{2}\right] \\
\operatorname{kill}[S] & =\operatorname{kill}\left[S_{1}\right] \cap \operatorname{kill}\left[S_{2}\right] \\
\operatorname{in}\left[S_{1}\right] & =\operatorname{in}[S] \\
\operatorname{in}\left[S_{2}\right] & =\operatorname{in}[S] \\
\operatorname{out}[S] & \text { PROF:Abutt[SiPABOut }\left[S_{2}\right]
\end{array}
$$

## Reaching Definitions


is of the form


$$
\begin{aligned}
\operatorname{gen}[S] & =\operatorname{gen}\left[S_{1}\right] \\
\operatorname{kill}[S] & =\operatorname{kill}\left[S_{1}\right] \\
\operatorname{in}\left[S_{1}\right] & =\operatorname{in}[S] \cup \operatorname{gen}\left[S_{1}\right] \\
\operatorname{out}[S] & =\operatorname{out}\left[S_{1}\right]
\end{aligned}
$$

## Example Reaching Definitions



## Using Bit-Vectors to Compute Reaching Definitions



## Computation of in and out sets ...for

 reaching definitionsfor all basic blocks $\mathrm{BB} \quad \operatorname{in}(\mathrm{BB})=\varnothing$ for all basic blocks $\mathrm{BB} \quad \operatorname{out}(\mathrm{BB})=\operatorname{gen}(\mathrm{BB})$
change $=$ true
while (change) do
change $=$ false
for each basic block BB , do

$$
\begin{aligned}
& \text { old_out }=\text { out }(\mathrm{BB}) \\
& \text { in }(\mathrm{BB})=\bigcup(\text { out }(\mathrm{Y})) \text { for all predecessors Y of } \mathrm{BB} \\
& \text { out }(\mathrm{BB})=\operatorname{gen}(\mathrm{BB})+(\text { in }(\mathrm{BB})-\operatorname{kill}(\mathrm{BB})) \\
& \text { if }(\text { old_out }!=\text { out }(\mathrm{BB})) \text { then change }=\text { true } \\
& \text { endfor } \\
& \text { idfor }
\end{aligned}
$$

## Example Reaching Definitions


$\operatorname{in}(\mathrm{BB})=\bigcup(\operatorname{out}(\mathrm{Y}))$ for all predecessors Y of BB
$\operatorname{out}(\mathrm{BB})=\operatorname{gen}(\mathrm{BB}) \mathrm{U}(\operatorname{in}(\mathrm{BB})-\operatorname{kill}(\mathrm{BB}))$

| Block <br> s | Initial |  | Pass1 |  | Pass2 |  |
| :--- | :--- | :--- | :--- | :---: | :---: | :---: |
|  | In | Out | In | Out | In | Out |
| B1 | 0000000 | 1110000 | 0000000 | 1110000 | 0000000 | 1110000 |
| B2 | 0000000 | 0001100 | 1110011 | 0011110 | 1111111 | 0011110 |
| B3 | 0000000 | 0000010 | 0011110 | 0001110 | 0011110 | 0001110 |
| B4 | 0000000 | 0000001 | 0011110 | 0010111 | 0011110 | 0010111 |



| Pass2 |  |
| :---: | :---: |
| In | Out |
| 0000000 | 1110000 |
| 1111111 | 0011110 |
| 0011110 | 0001110 |
| 0011110 | 0010111 |

Thus we can finally say that at the end of block 4 the final definitions reaching are<br>d3, d5, d6, d7

## Computation of in and out sets ...for

## available expressions

$\operatorname{in}(\mathrm{B} 1)=\varnothing$
$\operatorname{out}(\mathrm{B} 1)=\operatorname{gen}(\mathrm{B} 1)$
for $\mathrm{B} \neq \mathrm{B} 1$ do out $[\mathrm{B}]:=\mathrm{U}-\operatorname{kill}[\mathrm{B}]$
change $=$ true
while (change) do
change $=$ false
for $\mathrm{B} \neq \mathrm{B} 1$, do begin

$$
\begin{aligned}
& \text { in }(\mathrm{B})=\cap(\text { out }(\mathrm{P})) \text { for all predecessors } \mathrm{P} \text { of } \mathrm{BB} \\
& \text { old_out }=\operatorname{out}(\mathrm{B}) \\
& \text { out }(\mathrm{B})=\operatorname{gen}(\mathrm{B}) \mathrm{U}(\operatorname{in}(\mathrm{~B})-\text { kill }(\mathrm{B})) \\
& \text { if }(\text { old_out }!=\text { out }(\mathrm{BB})) \text { then change }=\text { true }
\end{aligned}
$$

endfor
endfer

## Live Variable (Liveness) Analysis

- Liveness: For each point p in a program and each variable $y$, determine whether $y$ can be used before being redefined, starting at p .
- Attributes
$-\boldsymbol{u s e}=$ set of variable used in the B prior to its definition
$-\boldsymbol{d e f}=$ set of variables defined in B prior to any use of the variable
- in = set of variables that are live at the entry point of a B
$-\boldsymbol{o u t}=$ set of variables that are live at the exit point of a B


## Live Variable (Liveness) Analysis

- Data flow equations:

$$
\begin{aligned}
& \operatorname{in}[B]=\operatorname{use}[B] \bigcup(\operatorname{out}[B]-\operatorname{def}[B]) \\
& \operatorname{out}[B]=\bigcup^{\operatorname{in}[S]}
\end{aligned}
$$

$-1^{\text {st }}$ Equation: a var is live, coming in the block, ifeither
-it is used before redefinition in B
or
-it is live coming out of B and is not redefined in B
$-2^{\text {nd }}$ Equation: a var is live coming out of B , iff it is live coming in to one of its successors.

## Example: Liveness



## THANK YOU!!!!!!!!!!!

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